Prof. Hans Jockers, Prof. Manfred Lehn

Oberseminar: Geometry & Physics
Introduction to Mirror Symmetry

Time: Mondays, 2:15pm–3:45pm — **1st meeting:** October 27, 2025

Place: Room 04-432

Mirror symmetry is a stunning bridge between mathematics and string theory. It predicts that certain complex and symplectic manifolds — called Calabi–Yau manifolds — come in mirror pairs, where the complex geometry of one relates to the symplectic geometry of the other.

In string theory, mirror symmetry emerged with the discovery in physics that two very different string compactifications can describe the same physical theory. This revealed deep connections between quantum phenomena and classical geometry.

Mathematically, it connects enumerative geometry, Hodge theory, and derived categories, while in physics it introduces a notion of quantum geometry and an example of a precisely formulated duality. From predicting curve counts to inspiring new categorical frameworks, mirror symmetry stands as a prominent example of how abstract mathematics and theoretical physics come together.

The aim of the seminar is to introduce the concept of mirror symmetry to advanced physics and mathematics students. On the physics side, we explore the emergence of mirror symmetry from Calabi–Yau compactifications of string theory, and on the mathematics side, we discuss the like between Hodge theory and enumerative geometry, which links complex structures, curve counting, and quantum phenomena in a single elegant framework.

Outline of Covered Topics

Supersymmetric Non-Linear σ-Models with Calabi–Yau target spaces
Definition of non-linear σ-model, Supersymmetry & Kähler manifolds, Axial Anomaly & Calabi–Yau manifolds [1, 2]

Counting Rational Curves in Calabi–Yau manifolds
Schubert calculus, enumerating lines & conics, stable curves and stable maps [3, 4]

Mirror Symmetry of Quintic Threefold
Example: Mirror Symmetry of the quintic Calabi–Yau threefold [5, 3]

Correlation functions in Non-Linear Sigma Models
A-type & B-type observables as cohomology elements of Calabi–Yau manifolds, correlators of A-type and B-type observables, mirror symmetric observables and correlators [2, 3, 5, 1]

Variation of Hodge Structures
Complex structure moduli spaces, variation of Hodge structures [5, 3]

Quantum Cohomology Ring
Definition of quantum cohomolgy ring & its algebraic properties [3]

References

[1] J. Distler, "Notes on N=2 sigma models," arXiv:hep-th/9212062.

[2] E. Witten, "Mirror manifolds and topological field theory," *AMS/IP Stud. Adv. Math.* **9** (1998) 121–160, arXiv:hep-th/9112056.

[3] D. R. Morrison, "Mathematical aspects of mirror symmetry," *Complex Algebraic Geometry (J. Kollár, ed.), IAS/Park City Math. Series, vol. 3, 1997, pp. 265-340* , alg-geom/9609021 [alg-geom].

[4] S. Katz, "On the finiteness of rational curves on quintic threefolds," *Compos. Math.* **60** no. 2, (1986) 151–162.

[5] D. A. Cox and S. Katz, *Mirror symmetry and algebraic geometry.* American Mathematical Society, 2000.